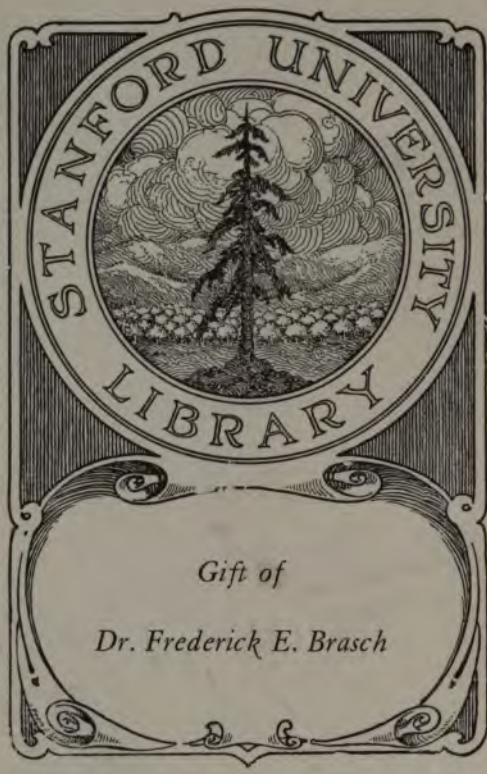
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Place of Mathematics in University Education.

INAUGURAL ADDRESS

OF

CHARLTON T. LEWIS,

PROFESSOR OF PURE MATHEMATICS IN TROY UNIVERSITY,

DELIVERED BEFORE THE TRUSTEES AT THEIR ANNUAL MEETING,

JULY 20TH, 1859.

PUBLISHED BY THE TRUSTEES.

TROY, N. Y.:
A. W. SCRIENER AND CO., BOOK, CARD AND JOB PRINTERS, CANNON PLACE.
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ADDRESS.

"Mathematics from the first have been triumphant over the husk; Philosophy is still militant for the kernel." Such is the judgment of Sir William Hamilton, one of the highest authorities, certainly, in any question which authority can decide; and among all thinkers our age has seen, one of the noblest inheritors of a yet unfulfilled renown. He urges as a leading and fatal objection to the use of Mathematics in education, that they deal only with the *husks* of truth. He would accordingly proscribe them, where mental culture is the aim. He further objects to this study certain special evil effects upon the mind. It is so true, so absolute and unquestionable in every step and every result, that it loses all the force of truth. It is so reliable in its methods, that it compels belief, and trains the mind to skepticism! So rigid and exacting in its proofs that it makes men credulous and superstitious! If all these sad and opposite results do indeed proceed from the study of Mathematics, we can wonder no longer that one tree is the tree of the knowledge of both good and evil. We must agree with one of the church fathers who referred this science to a very low, bad source indeed, and join the superstitious Banquo in saying,

> "'tis strange,
> And oftentimes to win us to our harm
> *The instruments of darkness* tell us truths;
> Win us with honest trifles, to betray us
> In deepest consequence."

But who can believe it? To all who love this science, to all whose eye is not evil through prejudice, Truth as revealed by Mathematics is Truth in her native beauty, with her disguises stript away; and thus seen, led out from the veils of form and matter behind which half her light is quenched, into the clear air of exact knowledge, she is the purest and sweetest "daughter of the voice of God;"—while, the more absolute the Truth, the surer is the signet of the Divine.

If then we free Sir William's aphorism, as we may, from the sneer about "husk and kernel," it means simply that "Mathematics have always been triumphant; Philosophy is still militant." Now this occurs in a piece of special pleading against Mathematics. He wishes to drive them out from the course of mental training utterly, reserving only those elementary branches which every educated man needs in daily life; and to substitute a course of metaphysical study. We are justified then in accepting these words as being, in the author's view, what they really are in themselves, an epitome of the history of these two sciences. But what a confession does the special advocate of metaphysics make, in his very plea! He would reject from education a triumphant branch, and would substitute for it a militant branch. He would supplant an established science by that which is only struggling to become a science. He would put away a method which has broken down a thousand barriers, each of which threatened to become the horizon of human knowledge forever, in favor of one which has never yet made one step of real progress, because its starting-point changes with every mind that attempts it; and whose chief contribution to science, an enemy might say, has been the multiplication of the known species of monomania!

Glance hastily at the history of the two. You see that Mathematics have been always steadily progressive; metaphysics have been fixedly stationary. The former

science has been adding at every step to human knowledge, in all directions, for it is thoroughly trustworthy, and what it does once is done forever. The latter has been actively moving indeed, with an infinite variety of motion, but in an orbit ever returning upon itself. The former accordingly has always been the wonder of the uninitiated; the latter their laughing-stock. You remember the judgment of Varro, Cicero's friend, and the greatest reader, as well as the most voluminous author, of ancient times. He assures us that there is no absurdity, capable of utterance in words, so gross that it has not been advanced and defended by some one of these self-styled philosophers. If this was true two thousand years since, what shall be said of the multiplied absurdities of modern metaphysics? What would Varro say, could he stand in the metaphysical alcove of a modern public Library? It was impossible that this waste of intellect and labor should take place without remonstrance from the true spirit of science; and in the present age, which exaggerates utility and aims only at a visible goal, it is impossible to keep this reaction and remonstrance within reasonable bounds. Hence modern science tends to ignore metaphysics and even deny their possibility. Indeed, their utter exclusion from the realms of human knowledge is a fundamental principle of the Positive Philosophy, and is being accepted more and more widely, under the vast and growing influence of that system. We may regret this movement, in view of the materializing influences, which, at best, have such power over contemporary thought; yet we cannot altogether blame it. For the method pursued by the metaphysicians can but appear to those whose habits of thought are formed amid the exact scientific methods by which other branches have been advanced, utterly and radically wrong. I confess that they seem so to me. The human mind has but one mode of discovering new truth; induction from particular facts. Yet the failure of the greatest minds which have devoted

themselves to Philosophy has not taught their successors to introduce that mode here. Hence the whole science still waits for its new language and its new method. Hence the melancholy spectacle presented by him from whom I first quoted; one of the most critical and brilliant intellects of modern times; who, if ever any man was so endowed, seemed to have burning within him a light to illuminate the whole realm of mental science; but almost shut it in upon itself, and was content to scatter here and there a bright ray in the dark corners of the theme. Hence too the sad fact which he urges on our notice; that while Exact Science has been steadily advancing, with *Let there be Light* upon her banners, and ever new worlds of daring thought adding their glory to her train, the halls of metaphysics stand to this day, a mere circus for the showy wrestling of mental athletes. The Mathematics have traced in the changeless laws of nature the image and superscription of the Immutable God, while the metaphysics, after so many ages of toil and wrangling, are yet without one general fact, one received doctrine.

Now in selecting a science for mental discipline, we need no great Philosopher to tell us that we must have one which has *results*. That one is most desirable whose methods are most perfect, and whose discoveries are most certain. We wish to accustom the mind to rely with well-grounded confidence on its own movements and its own conclusions. Only thus can the true scientific spirit be awakened. The young mind which learns to seek for truth by metaphysical methods, amid the surer training of this age, is sending his thoughts by a lame footman, while his neighbors use the lightning telegraph. He takes his place for his mental journeys on the old blind nag of Speculation, whom he leaves with loose rein to find her own way across the boundless heath; while the world is rushing and whistling past him along sure, prepared paths of iron demonstration, with the chained forces of

nature as attendants. At the peril of the young mind, its training must be something solid, something trustworthy. The early training of the scholar must teach him to use his energies in seeking firm and reliable truth, not in defending vague, verbal theory.

But though metaphysics be too uncertain to afford this training, will not Logic answer? It has often been suggested as a substitute for Mathematics, and as a better discipline for that scientific spirit, which is to be to the young student what his proboscis is to the Elephant; a test which he applies instinctively to his path as he advances, step by step; and which prevents him from passing a bridge of speculation to an island of mere theory. It might have been thought sufficient once; only a few generations ago, before the spirit of '76 awoke in science or in politics, and when all the schools occupied steps below the throne of Aristotle. But since, amid the dusty crowd and stormy wrangling of these latter days, Logic itself seems to have become too uncertain for elementary instruction. I mean, those principles, if any, which it possesses as established beyond controversy, and which may be given to the young as unquestioned truth, are too few, too incomplete, too dimly apprehended, to be employed as an instrument of early mental culture. Considered as the Science of Thought, it is terribly weak as applied to inductive reasoning; which indeed no one has ever yet succeeded in reducing to an acknowledged logical form, except by shutting up the whole difficulty of the argument in the major premiss, by an astonishing *petitio principii*. Yet take away Inductive reasoning from the domain of Logic, and what is left worth contending for? She may rule her little province as she will; beyond the notice of an inquiring, advancing age; and cut off from the fellowship of the sciences. They who wish to appreciate the uncertainty of Logic in its details may examine the controversy between Professor De Morgan and Sir William Hamilton, on the question whether a judg-

ment of identity between subject and predicate is simple or complex; a controversy which seemed to turn entirely on the difficulty of defining what is meant by a simple judgment. And the radical uncertainty of the basis on which the boasted science of thought rests, will be apparent to those who recall the fundamental variance between Sir James Mill and Archbishop Whately on the definition and aim of Logic itself.

But shall Philosophy be abandoned? Must we submit to the degrading doctrines now urged in high places, which limit all our knowledge to the world, and deny to the mind its God, its own inner life, its immortality? In the name of humanity, which Christian hope stamps as the one thing eternal amid a changing world, we ask, can there be no science of mind, of man? This hope for truth in these high themes where alone truth is worthy of our highest effort and sacrifice, the hope which has been the pole-star of earth's bravest and best minds, must we turn our backs on it forever? Philosophy! the gorgeous dream of Plato, the grim sport of Kant, the ideal love of Schelling, the dearest faith of Coleridge, is it all in vain? Philosophy! No, we stand in hope, and by the touch of her guiding hand and the cheering impulse of her voice, we know her near, and know that the crown is on her brow, and the glory in her smile; though our eyes lack the anointing, and cannot see. She leads us to the threshold of a temple vaster than the world, with room within for the expansion of the immortal mind, of the enthusiast's soul. She tells us that its shrines are rich with the spoils of infinite Beauty, and bright with the gems of enduring Truth; and that we may enter, and all this shall be ours, if we but read and translate into the language of life and heart two words which she has graven in the cold marble far above our heads. Painfully we climb; we feel for the letters like the blind; we trace in them forms we know; the Gamma, the Theta, the Sigma of our boyish tasks are here. The *words* are familiar;

every school-boy has read them; every language has adopted them as household words; but alas! they are for us words only; to the deep-searching eye of our guide, her ΓΝΩΘΙ ΣΑΥΤΟΝ is still unread; and we are still groping at the threshold.

But not forever. The way is preparing in the world of thought for a criticism as searching and thorough as Comte's; and far more comprehensive and catholic. It will make all science, as now established, a preparation, a propaedeutic, for a true Philosophy. The methods of investigation and discovery, pursued in other branches of knowledge, will emerge under its influence, purged from all that is accidental, or dependent upon their subject-matter, ready for a transfer to a higher sphere, and transfiguration by a higher truth. In the History and Biography of nations and individuals, in the Annals of Crime, in the dawning truths of Physiology, and in the Records of Lunatic Asylums, seem now to be collecting the masses of facts to which these methods must be applied; from which we may hope, perhaps, for laws as reliable as any known in the material world, and far more fruitful. But it must be acknowledged then that Philosophy is the pinnacle, not, as so many dream, the foundation of the Temple of Science; that she comes to us as the last child in the royal line of the Sciences, and the heir of all their wealth and power; not as the nurse and foster mother of them all. Yes, the way is preparing; the world is ripening for her sway!

> —"So runs my dream, but what am I?
> An infant crying in the night,
> An infant crying for the light,
> And with no language but a cry!"

But this I know, whether or not my answer be good and true, humanity's answer is sure. It speaks in the impulse within us, which will not let these themes rest, but drives us to pursue them. We feel our ignorance, therefore there must be a capacity for knowledge. There

can be no shadows in that land where no sun shines. That the needle of the mind's wild compass vibrates ever restlessly does not shew that there is no pole; but that there is a pole, though as yet undiscovered. Until now indeed, this crown science has opened to man only windows which look out upon an infinite dark; but they are windows still; and she will wait and watch through them till the morning dawns.

Meanwhile, until this Science appears as such among men, until its principles are established, and its outlines agreed upon, its study can have no place in elementary discipline. It has indeed a powerful awakening influence on the mind. Kant stirs the intellect as ambition stirs the heart. The student whose habits of thought are formed cannot read his works, or those of his fellow giants, without gaining in the perseverance, energy and clearness of his mental powers. But for the mind's early studies, which are not only to stir, but to direct it, something is needed in which all thought moves to useful ends. Else we shall have a mind, roused indeed to impetuous and violent struggling, but with no aim save the struggle itself; destitute of the scientific conception of truth, and perhaps doomed to endless speculation and hopeless distrust.

Granting the need, then, of rigid and reliable training, the question arises, how far can the Mathematics afford it? There are certain benefits which are so generally acknowledged to result from this study, that I need not dwell upon them here. Perhaps those least disputed and most frequently sought are its effects when used strictly as a *medicine;* a remedial agent for minds deformed or imperfect. Such are the uses mentioned by Lord Bacon; "If the wit be too dull, they sharpen it; if too wandering, they fix it; if too inherent in the sense, they abstract it." Perhaps the last of these, though the least noticed, is the most important and valuable. For this disease, "a wit too inherent in the sense," is not a

mere occasional disorder; but a congenital infirmity of human nature itself; never more dangerous, perhaps, than in our own age and nation. And nowhere besides has education an instrument of such power to remedy it as in Mathematics. The symbols of Mathematics are representatives of an idea which they always suggest, but which is not in them. The *wit inherent in the sense* must needs miss their meaning. They teach us to keep the mind's eye on that which the body's eye cannot see. And thus surely are we made and placed here to walk this world, with the inward eye on that which is behind and beyond the world; to see, through divine light reflected from our own spirits, everything tinged with brightness from the Creator's hand—

> "Earth crammed with heaven,
> And every common bush afire with God;"—

the whole visible creation a symbol, intricate but yet decipherable, of a Supreme Power. But we poor, blind children—we gaze at the stars, and miss the glory. We linger in the formula, and forget the idea. We cling closely to the folds of flesh, and lose, of the Spirit, even

> "Those shadowy recollections,
> Which, be they what they may,
> Are yet the fountain-light of all our day;
> Are yet the master-light of all our seeing."—

Into these things, indeed, the Mathematics give no direct insight; but they help the young mind, in forming its habits of thought, to emancipate its powers from the emblem, and direct them to the inner Truth.

Again, it seems to be unquestionable that this study fixes the attention; forms the habit of order in developing the student's own thoughts, and in presenting them to others; and disciplines the mind to rigid, entire impartiality. Every preconceived notion, every prejudice, must vanish when it meets Truth. Now in order to obtain these advantages in full measure, it is obvious that the study must be pursued in early life. The longer it is

postponed, the more difficult it becomes; and the more uncertain the benefit derived from it. A careful examination of the question, in the light of such facts as I have been able to gather, has led me to conclude that in our preparatory schools, and at an early age—from twelve to fifteen years, according to circumstances—nearly every child might and ought to master the principles of Arithmetic, Algebra, and Elementary Geometry, and acquire a moderate degree of practical skill in their application. This may be done, without in any way neglecting more general culture. There are objections, valid enough, against any approach to an *exclusive* cultivation of Mathematical Science. It does not awaken or develop critical taste, enthusiastic sympathy with noble sentiment, or the important but rarely matured power of balancing conflicting evidence. Hence the mental culture which is given by Mathematics alone is deformed and partial. This kind of one-sided development has sometimes, though rarely, occurred in our schools; under teachers who, from imperfect views of education or a peculiar mental bias, have given exaggerated importance to this study. But all such evils are sufficiently avoided by the adoption of two important practical principles; 1st, all early Mathematical study should be accompanied by parallel and at least equal attention to branches of a more literary character; 2nd, unless rare natural endowments justify it, no child under fifteen should be required to pursue this study beyond the elementary branches named above. To this last rule I ascribe great importance; believing that when the principles of the elementary Mathematics are fully learned, in their mutual dependence and relations, their properly educational value is exhausted; and that all time spent upon them afterwards, to gain a mere facility in wielding the machinery of operation, might as well, for purposes of general culture, be spent on the treadmill.

Indeed, it is an universal maxim, which has a thousand applications, that all true education combines, at each step,

new methods of operation with the attainment of new truths. Only by this combination can it secure to the young mind its native freshness, and give it harmonious development. Hence, whenever we cease to multiply the known paths in which the mind can move, and at the same time to widen its field of view, we cease to educate. If for this widening and elevating of the mind, we substitute, in any department, as the aim of instruction, mere dexterity in the use of its mechanism—the *professional skill*, whether of the mathematician, the artisan, or the critic—we degenerate from teachers of science to teachers of craft. In the thickest of the fight we turn traitors to our cause; and arm our advancing squadrons with the caduceus of Hermes instead of the Aegis of Athene; with the staff of a sorcerer, instead of the dazzling emblem of wisdom, and the flashing terrors of the serpent-crested Gorgon.

The elementary course of study should close, I think, with the fundamental branches named above. But the resources of Mathematics for general culture do not end here. True, literary students rarely pursue the study further. But this is because our schools are so far behind the demands of philosophical education. They do not accomplish half the task assigned them. Hence the Colleges are compelled to do this elementary work, which should have been done before admission. The student has now attained such an age, and such fixed habits of mind, that it requires a much longer time to gain a thorough knowledge of Algebra and Geometry than would have sufficed in childhood, with proper instruction, to do the very same work. Thus, even by the best scholars, the time which should properly be given to an extended course of study, is necessarily devoted to the elements; while many minds are so confirmed in unmathematical habits of thought by this time, that the science is really beyond their reach. Many Colleges have so far yielded to this state of things that practically they have aban-

doned Mathematics as a condition of a degree; and confer their honors freely on those students who have a tolerable knowledge of the Literary branches, to whom a quadratic equation is as intelligible as an Assyrian inscription; and who would slip as often and as hopelessly in a demonstration of Euclid, as in ascending Mount Ida on a rainy day. Hence it is that nowhere in America has a true University course of Mathematical study been inaugurated; and, so far as I can learn, the Troy University is the first institution in America to include such a course in its plan.

I will now attempt to outline such a course. Its object being to use Mathematical Science, so far as it can be made available, for mental culture; and as part of a broad and general scheme of education, intended to develop harmoniously all the intellectual powers; it leaves out of view entirely *special or professional training.* It aims, not to make Architects, Engineers, or Mathematicians, but to make men. Hence it views Mathematics, not as an instrument to be used, but as a science to be comprehended and understood. Assuming a thorough acquaintance with the elementary branches, which every student should acquire in childhood, the University course treats, in addition to the practical outline of the whole science and its applications, as usually afforded by the best instructors, of

I. *The Logic of Mathematical Reasoning.* The modes of inference and of investigation employed in this science must be compared with those in other branches of thought. Thus the student learns the degree and kind of credit due to its conclusions. This is indeed disputed ground, and many have supposed that the reasoning in the Mathematics is essentially distinct from that needed in other sciences and in practical life; so that the study of their Logic is of no value for the general culture of the reasoning powers. In opposition to this view, which, though rarely urged in a definite form, seems yet to be practically

gaining ground in some quarters, I assert that in Mathematics we have the *model* and the *ideal* of all reasoning; the perfect standard, which at once measures and limits the possible certainty of the mind's movement in every department of knowledge. There exists no analysis of Mathematical Logic on a satisfactory plan, so far as I can learn. Were such a work accessible, it would suffice to refer to it here. In its absence, it is of course not the occasion to attempt to supply it; but I will briefly indicate some general views, which will sufficiently support my claim for this Science.

We are assured that all possible inferences of the human mind are made in one of two forms or methods, called Induction and Deduction. But the distinction between the two is sometimes made very vague. Nor is it certain that in any sense different mental faculties are employed in them. The true distinction seems to be this; Induction rises from one or more individuals to a class containing them; Deduction descends from the class to the individual. Thus Astronomers, some two generations ago, observed a revolution about a common center of gravity in the twin stars Ξ in the Great Bear; and inferred that the law of Gravitation extends beyond our solar system, out across the inconceivable void around us, and governs the regions of the stars. This was an Induction. But if they had assumed, in the first place, that Gravitation is universal, and had thence inferred that the stars named revolve about each other, this would have been a Deduction. By a little reflection, we shall see that the former method alone can advance to truth that is reliable and wholly new; that is, only Induction *discovers;* the other properly evolves from a principle something already tacitly contained in it attaches a predicate that was before implied in the subject; transforms our knowledge, systemizes the results of Induction, and builds on *hypothesis*, in every department of thought, *theories* which usually fall of themselves, but sometimes become

the scaffolding for a great edifice of scientific Truth. Now that the study of Mathematics develops the deductive power, is not, I suppose, disputed. Its rare merit in this respect is that it compels a clear conception of premises, and a careful regard to all the elements of a problem, whether observed or assumed. This is surely the great want in the education of thinking men everywhere, for here their great fault lies. The strongest inducements to watchful accuracy are held out by a science which gives nothing without this care, but inevitably rewards it when exercised. The young Mathematician who, for want of this care, misconceives or misstates the least of all his premises, will soon feel in his wasted labor the punishment which the unrelenting Muse Urania inflicts on her unfaithful votaries; for his conclusions will be as valueless as those of the political speculators of Europe in 1848, when, in stating the conditions of their great question, they omitted the little incident, too trifling to attract their notice in marking out the great future of the nations, that the name and family of Napoleon had not quite vanished from the world!

But a common objection to Mathematical reasoning is that it is *exclusively* deductive, and therefore does not educate the inductive faculty. But I think a close analysis would shew that this view is based on a confusion of ideas. It is only possible here to indicate the proof in outline.

In every act of Inductive reasoning, there are two parts; a mental analysis of individual facts or objects, and a mental synthesis of that which is common to any number of them. Either of these parts may however predominate greatly; almost, indeed, to the exclusion of the other. Thus in determining from observation the general form of the planetary orbits, the analysis is the whole difficulty of the problem; the synthesis being obvious and unavoidable. In classifying plants or animals, on the other hand, the analysis is almost instinctive, while

the synthesis requires great diligence and caution. In any case, however, these two parts combine to form the reasoning process, and the certainty of the Induction depends jointly on the mental act and the subject-matter. The former, though requiring special care, according to the complication of the problem, is always essentially the same in its nature. But the latter varies illimitably. Yet amid all its varieties, the one thing essential to a valid Induction is obviously that the objects or individuals examined be a true representation of the class under consideration. The certainty of the inference does not depend at all on the number of instances examined, but solely on their representative excellence. Thus it often happens that a single instance, which is perfectly typical, is a better basis for an Induction than a thousand imperfect ones would be. If any astronomer in the world should distinctly detect a star remaining visible in the field of his telescope, a small fraction of a second after theory has placed it behind the moon's disk, this would prove refraction near her surface, and establish her atmosphere forever. Two careful experiments of Torricelli, one with water and one with mercury, determined the gravitation of the air for all ages. A few cautious analyses, by a reliable chemist, would fix, to the confidence of the scientific world, the combining number and leading properties of a newly discovered element. But almost boundless observation is needed to establish, if indeed such things ever are established, the nature of a new disease, or the value of a new remedy. And in fine, so numerous and intricate are the complications which invest the question whether the several races of men are true species, or varieties from a common origin, that, in a merely scientific point of view, it is usually considered as undetermined to this day.— Thus, when the facts and principles of any science are harmonized, and have reached something like complete expression in a general law, universal conclusions may

often be deduced from one or a few observations; but when there are no such ultimate facts, or when exceptions are still frequent and unexplained, all such conclusions would be wild and worthless. In the former case, we are sure the *class-type* is before us; in the latter, we have no certain means of determining whether it is represented in what we see.

Hence the first question in examining any Induction is, how far do the examples adduced represent their class? Now turning to Mathematics, it appears that here, in many instances of reasoning, the very same mental process is employed of which I have spoken; but the subject-matter to which it is applied is entirely different. It is peculiar in this, that *each* instance is a perfect type of its class, and supports an Induction as conclusively as any greater number. As its laws are perfect in their generality, admitting of no exception, there can be no place for error. When the Mathematician examines an Isosceles triangle, he, by analysis, separates mentally just those features which are essential to constitute it one of its class, and confines his attention to these. Hence he cannot doubt that his demonstration of the equality of the angles opposite the equal sides is universal in its application to all such figures. This very simple instance, in which the mental analysis, and the formation of the class, are so easy, seems to shew that the element of certainty, so far as it distinguishes the Mathematics from other sciences, is not in the mental process which they require, but in their subject matter. If, in any other branch of knowledge, the observer could be sure that he has before him the perfect *class-type* he seeks, his Induction would become as necessary and certain as that of the Mathematician.

So far then as the pure logical movement of the mind can be separated from all that is unessential, the limitations and forms of its subjects, and presented as a model and ideal for all reasoning, this is done in Mathematics.

But the error I combat has arisen in part from the very fact that the reasoning is thus abstracted. In the common instances of Induction in science, there is very little reasoning done. If all the steps of pure logical sequence, involved in any common scientific research, and essential to it, were severed from the work of observation and experiment, of hand, eye and memory, and embodied in some symbolic form, like that of Geometry or Algebra, it would appear trifling indeed. Success in the experimental sciences, except for the very highest order of investigation and discovery, demands rather the skill of the observer and the mechanic, than great reasoning power. Now the Mathematics do not cultivate the eye and the hand for experiment and observation, or do so only subordinately and incidentally. Hence perhaps in great measure the confusion of thought which has represented them as the foes of Inductive Reasoning.

II. The second branch peculiar to the University Course is the *Philosophy of Mathematical Science.* This strictly includes its Logic, but comprehends besides the relation of Mathematics to the other Sciences; the extent of its applications, as conceivable, possible, and actual; and the classification and relations of its several branches. In all these points of view, this Science is more important than any other; from the higher perfection which it has attained, which renders its Philosophy the model and key to that of each of the other Sciences. It is of course impossible even to outline this theme to-day. I can only allude to one important branch of the Philosophy of Mathematics, that indeed by which the progress and perfection of this, as of every Science is best tested and measured, the study of its Nomenclature. The best possible proof that the true scientific spirit controls a science, and that the true era of progress is inaugurated in it, is that its Language is made consistent, uniform, and the exact representative of its fundamental ideas. By the terms which it contributes to our means of expression, among the most solid

and wide-spread results of its discoveries, does the world learn what it has accomplished. In an advanced science, each important word has a history of its own, and may embody in its inner life the struggles and conquests of a generation. This is true to some extent in social and political history. Words come down to us out of the past, for each of which blood has flowed, to each of which nations have contributed. The word *Liberty* has reached the meaning it has for us now, only after struggles and trials hard to comprehend; and who hears it in America to-day but seems to hear in it the rolling drums of the revolution? Who sees it on the page, but seems to see it tinged red with martyrs' blood? But this seems still more strikingly true in science. The word *Gravitation* recalls the history of Astronomy through her most brilliant period. The words *Undulation* and *Atom* are still the standards around which gather the conflicts and labors of modern Optics and Chemistry. When a great word is at length won, when it comes to our lips packed with the wealth of meaning which a past of thought and toil has given it, we, in using, in uttering it, are wielding the power of that thought, the strength of that toil. He who calls Americans to patriotism by their watchword of *Liberty*, gathers in it the history of our land, and through this representative word, the sacrifices and sufferings of our early days, quicken the heart-beat and stir the life-blood of children's children.—He who employs a scientific term, expressing or implying some great general law, really brandishes a Damask blade of thought forged for him amid roaring flames of toil by giant arms of older days. I never hear the word *Gravitation*, but the mind of NEWTON seems speaking to me out of a buried world.

Now this law of human progress, which embodies all principles, as they emerge into our knowledge, in living words, in utterances and symbols for eye and ear, and thus teaches the child of to-day the familiar use of truth which cost life-labor to the sage of former times; this

mode of advance is most easily and clearly traceable in Mathematics; the science of all others in which Thought is most inseparably blended with Language, and most dependent upon it. Language as such holds a higher place in Mathematics than in any other science. I exclude from view of course Logic, which is properly not a science, but a discipline for all sciences; which, however, as usually taught, seems to be purely verbal, and to bear to science the same relation which rope-dancing bears to navigation.

By Language in Mathematics I mean its own dialect; not words but symbols; all its forms of expression, indeed, whether of quantity or operation. It is peculiar in this, that it is strictly adequate to the conceptions it embodies. The ideas and processes of the mind are here expressed completely, without want or excess. Each new conception is expressed by a new term. Every new process demands a new symbol. Each of these conveys always the same notion to every mind which uses or reads it. It is impossible that those who have not made it a special study should know the extent, compass and uses of this language. It is absolute in its precision, entirely philosophical in the relations of its parts and terms, and indefinite in its capacities for extension. In all these respects, it very far excels every other means of expression known to the human mind. Its wonderful power is the result of vast accumulations of thought through many ages. Its excellence for its own purposes would be seen by any one who should attempt to translate from it into another tongue. The simplest transformation of an algebraic formula might possibly be put into English, by great labor, to the satisfaction of the experimenter; but even this he could scarcely hope to make intelligible to others. Pass to the splendid achievements which have given immortal fame to the representative minds of the modern Geometry. We find them moving from principle to principle, aye, from star to star, stride by stride along

a path of daring thought, on which not one step could the mightiest human intellect accompany them, unsupported by this instrument. A great educational work, then, is accomplished, if this vast wealth of Language is thrown upon the mind, not to be used as a mere machine, but to be studied and understood in its secret mechanism; to be measured, criticised and enlarged by the mind itself. Perhaps no more powerful discipline of the pure Inductive faculty can be imagined than is often the discovery of mathematical expression and form for a new class of problems. And what special branch of equal extent can be introduced into the University Course of study, more beneficial to the ordinary mind than the Philosophy of this wonderful language, the truest home of the power of modern Science? What will produce more surely the mental habit of exacting severe precision in defining and distinguishing terms? I think, indeed, it is not going too far to say that we cannot entirely sever the moral from the scientific idea of Truth; and that the study and use of such a language as this tends to produce a habit of aiming at strict correspondence between thought and expression, essence and form, fact and language, everywhere.

III.—Finally, the University Course includes the History of Mathematics, which combines the former branches, and completes the study of the Science. As a part of general human History, or even of the History of Civilization, this specialty has of course but a very subordinate position. But its simplicity, completeness and steadfast progress, must ever preserve to it the first place in the History of the Sciences. I am inclined to think that something of the historical method may profitably be introduced in teaching even the elements of Mathematics. Arithmetic is, to many children, a dry study, but is susceptible of illustrations from this source, which would often render it agreeable. No other branch of knowledge which the very young mind is capable of comprehending reveals so simply and impressively the great law of *ac-*

cumulation—the rule so sovereign in all History, individual and national, religious and scientific, *to him that hath shall be given*—as the records of the progress of Arithmetic. This method too adds great interest to all the details of the study. The child of eight or ten years old feels much more interest in his long question in division, when told that what he does in fifteen minutes would have been to Plato a long and tedious labor; and he has thus learned a little lesson in historic growth besides. The school-boy, familiar with figures, is delighted to learn that Euclid and Archimedes had been dead fourteen centuries, when notation by tens was introduced into Europe; and the mysterious cipher, which means *nothing*, and yet is represented by the circle, the emblem of the infinite, appeared in the books and frightened all the Alchemists. A still stranger lesson is given when he is told that three hundred years more elapsed before the brilliant invention of the decimal point almost doubled the power of numbers, and brought arithmetic nearly to its present form. And perhaps no better instance occurs anywhere of the modern encroachments of the scientific spirit upon the purely literary training of the middle ages, than the fact that in Shakspeare's time arithmetic was considered in England so abstruse and difficult, as to be reserved for the University Course, while boys were writing Latin and parsing Greek in the preparatory schools.

Indeed, the advances of this simple science afford the best possible type of the great progressive movement of the world. All its History, too, has left its traces somewhere among us. The old Greek Arithmetic was fanciful and dreamy; its methods were imperfect, weak and slow; and it felt its incompetency for the great work to be done, and turned aside to give its strength to vague notions about perfect and symbolic numbers. Yet even this early and superstitious Arithmetic has foot-prints among men to-day. All its thoughts are driven out long since from the schools, from education, from practical life. But they

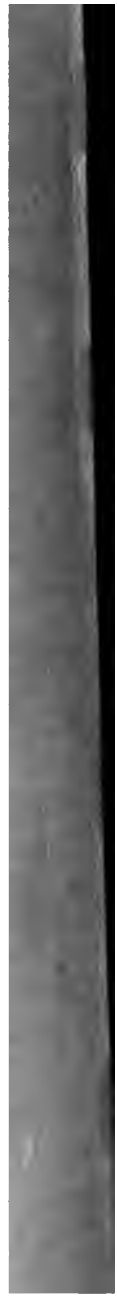
still find a refuge, with a thousand other exploded superstitions, in the laws which govern us, and the customs of society. Thus, Pythagoras found that *three*, which includes the beginning, the middle and the end of everything, is the most perfect of numbers; that *seven*, which was the number of known bodies in the solar system, was the number of the divine; and inferred that by multiplying the two together,—three times seven are *twenty-one*—he would obtain a number containing both the former, and therefore uniting their perfections. Age after age has passed by, and still, in many of the United States, contracts are regulated, and convicts sentenced under the law, by periods of *three*, *seven*, and *twenty-one* years! Nay, even our colleges bow to Pythagoras, and require an apprenticeship of just *seven* years from every pupil before he can become Master of Arts; while for the same all-sufficient reason, our great nation itself requires *twenty-one* years waiting of her children before they can claim the rights of citizens. Back through twenty-four centuries I turn, in the name of the young men of my audience, and bless the Sage of Crotona that he knew no more; for if, instead of the seven bodies in his solar system, he had known Uranus and Neptune, the seven moons of Saturn and the four of Jupiter, not to mention the fifty Asteroids, what would now be the length of Young America's probation?

But I am digressing. Suffice it to have indicated the fact that the History of Mathematical Science is a most fruitful theme in education; that, in short, the proposed University Course, embracing the Logic, the Philosophy and the History of Mathematical Science, will greatly benefit all students, and contribute to their completeness as educated men. This general view of Mathematics, as a science rather than as an art, seems to me at least equally desirable for those who intend to devote their powers professionally to it, or to some of its applications. But they need much more than this; a special training

for the work they have in view. Of this I have not spoken, as it is less suited to the comprehensive character of the Institution, and to a representative occasion like this; and although such special schools for Mechanics, Engineering and Surveying are included in the plan of the University, it is not proposed to begin them immediately. Whenever they shall be founded, and to whatever extent the Mathematics be pursued meanwhile within her walls, it will steadfastly be the aim of this Institution to cultivate the science in a catholic spirit, in view of its relations to all Truth; and to save it from the reproaches so often and so justly cast on its exclusive study. Thus pursued, it will not shut in the mind from the world and from the Light of Life, but will point the way to these. We are sometimes told, indeed, that the nature of Mathematical study is too remote and abstract; that it talks of straight lines and circles, lines mathematically straight, and circles of perfect roundness, of which the world knows nothing. I will not say, so much the worse for the world; but simply ask you to look up to-night to the sky, and see yon quiet star, not half so bright as the brightest in the sky, but still brilliant. The ray of light from it to your eye moved for twenty years, with inconceivable swiftness, in a line as straight as thought; without varying a hair's breadth in its course, until the thousandth of a second before it met your eye, when it struck the atmosphere of a fallen world, and was bent into the likeness of our universal crookedness. But straight lines were the theme of Geometry ages before the path of the ray was thought of. So the conic sections were traced out in the minds of the Greek Geometers, and their properties discovered and taught; and many centuries after they were found to be traced through the infinite heavens by vast globes from the Creator's hand. Thus when rightly and reverently studied, this science has ever led, and still will lead, to nature and to God. We will strive, in the name of Truth, so to teach it and so to learn it

here; avoiding the error into which some have sadly fallen who yet rank among the greatest names of the Science; who have rested in the certainties of abstract knowledge, and have forgotten the throbbing heart and varied life, the voice of duty and the sympathies of man;—

"Seeing not
That Beauty, Good and Knowledge are three sisters
That dote upon each other, friends to man,
Living together under the same roof,
And never can be sundered without tears.
And he that shuts Love out, in turn shall be
Shut out from Love, and on her threshold lie
Howling in outer darkness. Not for *this*
Was common clay ta'en from the common earth,
Moulded by God, and tempered with the tears
Of angels to the perfect state of man."



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